SDP Memo 061: SDP Bandpass Calibration Requirements during EoR Averaging

Document number…………………………………………………………..……SDP Memo 061
Document Type………………………………………………………………………….MEMO
Revision……………………………………………………………………………...……….01
Author………………………………………………………………………………..….D. Mitchell
Release Date……………………………………………………………...…….YYYY-MM-DD
Document Classification………………………………………………………….... Unrestricted

Lead Author:

Name	Designation	Affiliation
Daniel Mitchell	Senior Research Scientist	CSIRO
Signature & Date:	daniel.mitchell@csiro.au	

Reviewed by:

Name	Designation	Affiliation
Tim Cornwell	Reviewer	Tim Cornwell Consulting
Signature & Date:		

SDP Memo Disclaimer

The SDP memos are designed to allow the quick recording of investigations and research done by members of the SDP. They are also designed to raise questions about parts of the SDP design or SDP process. The contents of a memo may be the opinion of the author, not the whole of the SDP.

Table of Contents

List of Tables

List of Abbreviations

- CD: Cosmic Dawn
- EoR: Epoch of Reionisation
- RMS: Root Mean Square

Introduction

This memo provides background to the choice of parameters specified in two SDP L2 requirements: SDP_REQ-871 (Residual Phase Shift) and SDP_REQ-872 (Residual Amplitude). These parameters specify the bandpass calibration accuracy required before visibilities are average in frequency and exported to SKA Regional Centres for further processing by science teams (in particular EoR/CD). See SDP_REQ-858 (Time and frequency averaged visibility data) and SKA1-SYS_REQ-3605 (EoR/CD pipeline handover point).

References

Reference Documents

Reference Number	Reference
RD01	C. M. Trott and R. B. Wayth (2016) "Spectral Calibration Requirements of Radio Interferometers for Epoch of Reionisation Science with the SKA", PASA, 33, e019.
RD02	R. Braun (2013) “Understanding Synthesis Imaging Dynamic Range”, A&A, 551:A91.

SDP_REQ-871 Residual Phase Shift

Description: When SDP averages visibilities to wider frequency channel widths, variations in the phase error of station-based gains across the average shall not exceed 1.3 degrees.

Rationale: When subtracting a 5 Jy point source a phase gradient of this order will leave a residual with the expected strength of an ionised EoR tomography bubble. Trott & Wayth [RD01] consider a set of calibration models, ranging from optimistic to pessimistic. Here, we follow the pessimistic approach, which assumes that station-based phase errors dominate the residual, and that these errors are independent between stations and constant in time. Note that SDP is unable to correct any direction dependence in the phase gradients at this point, except during peeling.

Trott & Wayth [RD01] consider the effect on EoR tomography images of a residual phase shift across the 4.58 kHz channels. The same analysis can be used to set limits on acceptable bandpass calibration errors at the point that the SDP averages visibilities in frequency. Suppose that the stations have independent bandpass calibration phase errors that are approximately linear across the averages, with gradients that are normally distributed with an RMS of $2\pi\,\delta\varepsilon$ radians per Hz. If visibility phase gradients have an RMS $\sqrt{2}$ times as large, then averaging a given visibility would lead to decorrelation of the signal by

$$\int_{-\delta\nu/2}^{+\delta\nu/2} exp\left(2\pi\sqrt{2}\,\delta\varepsilon\,\nu\right)d\nu \;=\; sinc\left(2\pi\,\delta\varepsilon\,\delta\nu\right)$$

As discussed in [RD01], there is a statistically significant chance that a 1 degree ionised bubble will be contaminated by a 5 Jy source, while on degree scales the required noise floor of 1 mK is equivalent to 0.2 mJy. As it is assumed that the phase errors are constant for the duration of the integration, a dynamic range of 0.2/5000 requires that $\delta\varepsilon\,\delta\nu$ is less than $3.5\text{x}10^{-3}$ wavelengths – a station phase RMS no greater than 1.3 degrees.

Will SDP achieve this accuracy? Suppose that we are peeling a 5 Jy point source after station-based gain calibration and before averaging in frequency for output to an EoR Regional Centre. Following Braun [RD02], let the system temperature and effective area of LOW stations be

$T\_sys = \;150 + 60\,\lambda^{2.55}\,K$ and
$A\_eff = 2 \times 256\,\lambda^{2/3}\,m^2$,

such that the noise for a visibility sample with channel width $\delta\nu$ Hz and integration time τ seconds has an RMS of

$$\sigma_{vis} \;=\; \frac{2\,k_B\,T_{sys}}{A_{ee}\sqrt{2\,\delta\nu\,\tau}} \times 10^{26}\,Jy.$$

If the channel width is 5 kHz and the integration time is 5 minutes, then at 50 MHz σ_{vis} will be approximately 5 Jy. A source with a flux density of *S* = 5 Jy at 150 MHz will in general be closer to 11 Jy at 50 MHz, and the phase noise RMS in station-based gain estimates will be approximately $\sigma_{vis}/(S\sqrt{N})$ ≈ 1 degree for N=512 stations. At 250 MHz the visibility noise RMS will be just under 1 Jy and the source flux density will be closer to 3 Jy, and the phase noise RMS in station-based gain estimates will be approximately 0.5 degrees. These are within the required level of accuracy, and could be improved by an interpolation across the channel being averaged.

On faster time scales we can expect ionospheric phase fluctuations. Kolmogorov turbulence in the ionosphere with a diffractive scale of r_0 m leads to visibility phase noise with a variance of $(r/r_0)^\beta$ rad^2 at 150 MHz for a baseline of length r metres, where β=5/3. The phase increases as a function of wavelength, and the gradient across a frequency channel being averaged will be approximately $(\lambda_1\text{-}\lambda_2)/\lambda_{150MHz}(r/r_0)^{\beta/2}$ rad. For a relatively extreme diffractive scale of 5km, the RMS of phase shifts across a 100 kHz bandwidth at 50 MHz for 1 km baselines (i.e., in the EoR core) will be approximately 0.1 degrees. The phase shifts will not be independent across the core, however with a baseline RMS of around 400 m the average will closer to 0.04 degrees, or ≈ 1 degree if multiplied by $\sqrt{N}$. So it should be OK to leave direction-dependent ionospheric phase corrections until EoR processing at the Regional Centres. But if need be, low-order functions can readily be fit to the phases across the core and incorporated into peeling to decrease both the size of the phase shifts and the correlation between visibilities.

SDP_REQ-872 Residual Amplitude

Description: When SDP averages the visibility data to wider channel widths, the error in the bandpass correction (assuming a third order polynomial in frequency across ~ 300 kHz) must be <2.5% (50 MHz), <0.5% (150 MHz), <0.8% (200 MHz), as described in [RD01].

Rationale: This is the level of bandpass calibration across each 109.8 kHz science channel required for the EoR power spectrum measurement to be noise limited (as described in [RD01]). Note that SDP is unable to correct any direction dependence in the phase gradients at this point, except during peeling.

Trott and Wayth [RD01] consider how good the bandpass needs to be across each 109.8 kHz science channel for the EoR power spectrum measurement to be noise limited. For each of these science channels they considered polynomial fits to the 4.5 kHz fine channels of the science channel and its two neighbours. The residuals are correlated and lead to structure in the power spectrum with an amplitude that increases as the order of the polynomial is increased. So the allowable residual power level is dependent on the order required for the fit. Assuming a third order fit, the fractional amplitude of the fourth-order residual must be <2.5% (50 MHz), <0.5% (150 MHz), <0.8% (200 MHz). For 2nd, 3rd and 4th order fits the residual fractional amplitude limits are:

Band	δn=2	δn=3	δn=4
50 MHz	0.027	0.025	0.019
100 MHz	0.011	0.010	0.008
150 MHz	0.006	0.005	0.004
200 MHz	0.009	0.008	0.006

Table 1: Residual fractional amplitude limits for different order polynomial fits at four different frequencies.

As discussed for SDP_REQ-871, the error in the station-based gain solutions for a 5 kHz channel, 5 minutes of averaging and a 5 Jy calibrator, $\sigma_{vis}/(S\sqrt{N})$, will be approximately 0.02 at 50 MHz and 0.01 at 250 MHz, before any interpolation across frequency.